

# Time Domain Identification of PWM Converters for Digital Controllers Design

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**Abstract** -A discrete time-domain based system identification method for PWM DC-DC converters is presented. The proposed procedure is capable of successfully reconstructing the system's model from an arbitrary excitation at the command input. In this study, a step perturbation was applied, which is simple to apply and has an intuitive interpretation of the output response. The effects of switching and quantization noise were overcome by choosing the sampling instance to be after the switching oscillations decay significantly and by averaging the responses of synchronously perturbed sequences. The proposed method was evaluated on Buck and Boost converters. The digital data acquisition procedure was implemented on a TMS320F2407 DSP core. Excellent agreement was found between simulations and experimental results.

## I. INTRODUCTION

One significant source of inaccuracies in controller design is insufficient information of the open-loop response of the plant. This is particularly true in PWM converters, where uncertainties of the system parameters (load range, components spread and parasitics) often occur. The problems that stem from a poor knowledge of the plant become even greater when designing a discrete domain controller since additional error sources such as sampling, quantization and computational delays are present. After designing a digital controller, and even evaluating one by simulation, it is not uncommon to end up with poor closed-loop performance of the physical system due to inaccurate modeling of the system's plant. It appears then that it would be highly advantageous to base the design of a digital controller for PWM converters on a system identification procedure to obtain a realistic sampled data model of the system's response in open-loop.

Two general approaches have been described for digital compensator design for PWM converters. The most popular one is based on the frequency domain [1], [2]. In this case, an analog controller is first derived and is then transformed to the discrete form by one of the continuous domain to the discrete domain transformations, such as ZOH, p-z matched, etc. [2]. The second approach for discrete control design applies a

direct digital design procedure [3], [4]. This method bypasses some approximations and errors related to s-to-z conversion and can thus be considered to be more accurate. The accuracy of the design by this approach could be further improved if knowledge of the plant is extracted from experimental data of the system, i.e. by system identification.

Generally, system identification can be divided into parametric and non-parametric methods [2]. Traditionally, non-parametric methods were used to compute the system's frequency response from the results of either correlation analysis [2], [5] or spectrum analysis [2]. On the other hand, parametric methods [3], [6] pre assume the system's model (i.e. template) and the identification procedure is used to extract the parameters' values. Correlation-based identification often requires long data acquisition sequences to assure data accuracy and noise immunity, and additional manipulations of the data records (by cross-correlation and Fourier transform). The latter is potentially a cause for inaccurate model extraction due to truncation and quantization errors, and the approximated nature of the s to z transformations. To avoid these deficiencies, it is desirable that the identification procedure will be based on short data acquisition sequences and implemented directly in the discrete domain. These attributes are found in the discrete time-domain based parametric identification method that was proposed by Steiglitz and McBride [6] and has been applied on linear (non-switching) systems and on simulation models of grid power systems [7].

The objective of this work was to develop an identification procedure for modeling the open-loop response of switching converters. The motivation for this effort is the fact that the extraction procedure uses the system's input and output data records in the sampling (time) domain which does not involve additional transformations that may affect the model accuracy and that it utilizes a relatively small number of samples for model reconstruction. This identification approach, combined with direct digital control methods [4], can lead to very accurate and fully automated on-line, controller design.

## II. SYSTEM IDENTIFICATION ALGORITHM

The identification procedure for PWM converters that was developed in this study is based on the parameters extraction concept implemented in the MATLAB 'stmcb' command which is based on Steiglitz and McBride [6]. The most significant advantage of this method is that it uses the time domain data of the input and output records to extract the systems parameters. Below we present the essentials of the method.

It assumes that for every linear single-input single-output (SISO) sampled-data system, the relationship between the input and output records can be represented by a rational division of polynomials of  $z^{-1}$

$$\frac{N(z)}{D(z)} \quad (1)$$

where  $N(z) = a_0 + a_1 z^{-1} + \dots + a_{n-1} z^{-(n-1)}$  is the system's numerator and  $D(z) = 1 + b_1 z^{-1} + \dots + b_n z^{-n}$  is the denominator, which often referred as the systems characteristic equation.

The essence of the method is described in Fig. 1. It applies an iterative least square minimization procedure to extract the coefficients  $\{a_0, a_1, \dots, a_{n-1}, b_1, b_2, \dots, b_n\}$  such that the response of  $N(z)/D(z)$  ( $\hat{y}$ , Fig. 1) to the input records ( $u$ , Fig. 1) will match the actual (measured) output samples ( $y$ , Fig. 1).

Further details of the identification procedure are given in Fig. 2. It is based on the assumption that the system's order is known. This can be accomplished either by inspection or an a priori knowledge of the system dynamic characteristics. Secondly, a 'first-estimate' of the coefficients is calculated using the Prony estimate [8], yielding  $D_1(z)$  and  $N_1(z)$ . This method finds the filter coefficients by solving the linear regression on the input and output records. Finally, once the 'first-estimate' is set, then, for each step, the previous extracted characteristic equation ( $D_{i-1}(z)$ ) is used to prefilter the data records deriving new set of input and output records ( $\hat{u}$  and  $\hat{y}$ , Fig. 2) to be used in the minimization process

This identification calculation is implemented in the SM function of the signal processing toolbox [9] of MATLAB. It should be noted that the extraction of the 'first-estimate' is already imbedded in the algorithm.

## III. PROPOSED IDENTIFICATION METHOD FOR PWM CONVERTERS

All three basic topologies of DCDC converters (Buck, Boost and Buck-boost) can be modeled by a unified template, small-signal continuous form model [10]

$$A(s) G_{DC} = \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_0} + \frac{s^2}{\omega_0^2}} \quad (2)$$

where  $G_{DC}$  is the steady state gain,  $\omega_z$  is the frequency location of the systems zero,  $\omega_0$  is the natural frequency and  $Q$  is the converter's quality factor.

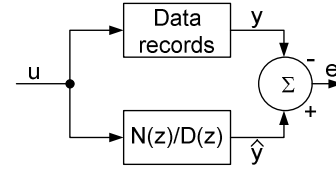


Figure 1. Identification objective.

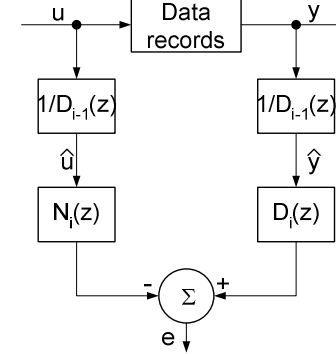


Figure 2. The SM identification procedure.

Analogously, applying p-z matching transformation, the unified discrete domain template will be [3]

$$A(z) = \frac{az + b}{z^2 + cz + d} \quad (3)$$

where  $\{a, b, c, d\}$  are the parameters to be estimated.

In CCM operation, the Boost and Buck-boost topologies are considered to be stable and non-minimum phase systems. That is, they include two left half plane poles and a right half plane (RHP) zero, while the Buck converter will have two poles and LHP zero and is therefore regarded as a minimum phase system. In other words, all three topologies share the same model template with different coefficients. This attribute simplifies the implementation of the identification algorithm by making it general and topology independent.

The discrete form of the small-signal transfer function given in (3) describes the response of the output signal of a converter ( $v_{out}$  in present study) to impulse perturbation of the control command (duty cycle, d). All for a given operating conditions. However, in practical applications, such disturbance is not practical to apply due to the infinite magnitude of the delta signal. Fortunately, the SM algorithm is capable of reconstructing the discrete time filter response from any type of disturbance, as long as the relation between the input and output records satisfy (1). This enables us to apply a more convenient and intuitive approach such as a step perturbation of the duty cycle command.

### A. Simulation-based identification of PWM Converters

To verify the identification concept outlined above, it was carried out on data that were collected by average model simulation [11]. The following steps were applied.

1. An average modeled Buck stage (Fig. 3) was subjected to a unit step. The parameters of the Buck were:

$L=75\mu\text{H}$ , ( $R_L=150\text{m}\Omega$ ),  $C=100\mu\text{F}$  ( $\text{ESR}=300\text{m}\Omega$ ),  $V_{\text{in}}=15\text{V}$ .

2. Data records of  $V(\text{out})$  (Fig. 3) and  $V(\text{Don})$  (Fig. 3) were collected in fixed sampling intervals ( $20\mu\text{s}$ ) to emulate A/D operation. This was accomplished by the print-to file option of SPICE (prn element, Fig. 3).
3. The samples were then used in a MATLAB identification procedure  $\{\text{stmcb}(n,d,it)\}$  [9], 'n' and 'd' are the order of the numerator and denominator respectively and 'it' is the number of iterations to be preformed.

The resulted discrete time plant model was found to be

$$A(z) = \frac{0.677z + 0.2683}{z^2 + 1.852z + 0.881} \quad (4)$$

Fig. 4 shows the very good agreement obtained when comparing the results of the step response of average simulation model to the response of the identified system (4).

### B. Practical implementation of the method

In order to implement the identification method experimentally on PWM converters, there is a need to reduce the effects of switching noise and parasitic ringing. This can be accomplished by setting the sampling instant to the end of each switching cycle, after the ringing related to the off transitions decay significantly (Fig. 5a). Another measure that is proposed for the reduction of noise interference and hence increases the accuracy of the measurement is by synchronous averaging of repeated perturbations, allowing the system to stabilize before the next excitation (Fig. 5b). The improvement of the signal to noise ratio obtained by this procedure is due to the fact that the converter can be considered 'time-invariant' and will thus exhibit identical responses to the same disturbances over time, while the noise will averaged out.

Additional issue that needs to be resolved is the magnitude of the injected step. Ideally, the size of the injected perturbation should be kept as small as possible. However, due to practical limitations of A/D conversion, a compromise between the measurement resolution and SNR, and the step size must be reached. The size of the step disturbance should be selected such that: (a) will not move significantly the operating point, but (b) excite a sufficient change at the output to allow reliable measurement.

A sequence of simulations was applied on the SM MATLAB function to assure convergence of the estimated coefficients and in attempt to find the optimal number of iterations needed for successful model reconstruction. This was accomplished by applying the generic template of (3) as the source transfer function, subjecting it to a unity step, and applying the data records to the MATLAB command. The model order to be found was kept constant, while for every run the number of iterations to be preformed was increased until the extracted coefficients were within two percent margin (or less) of the original values. This experiment was run for forty different model coefficients (twenty stable minimum

phase systems and twenty stable non-minimum phase system, all having same template). Convergence was obtained in all runs. It was found that given the second order template of (3), the optimal number of repetitions needed for the 2% accuracy was four for the minimum phase systems, and seven for the non-minimum phase model.

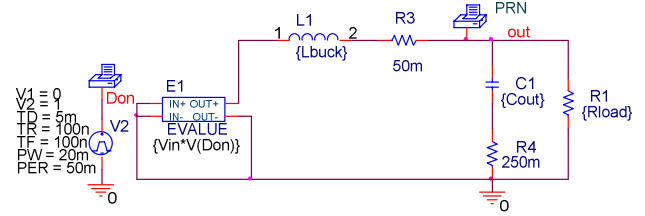


Figure 3. Average model of Buck type converter. Data records are sampled in fixed time intervals

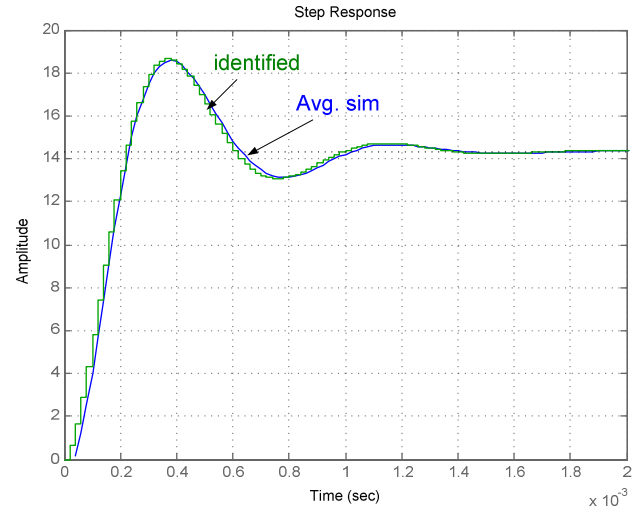


Figure 4. Step response of the identified Buck converter (Eq. 4) and original records obtained from average simulation model of Fig. 3.

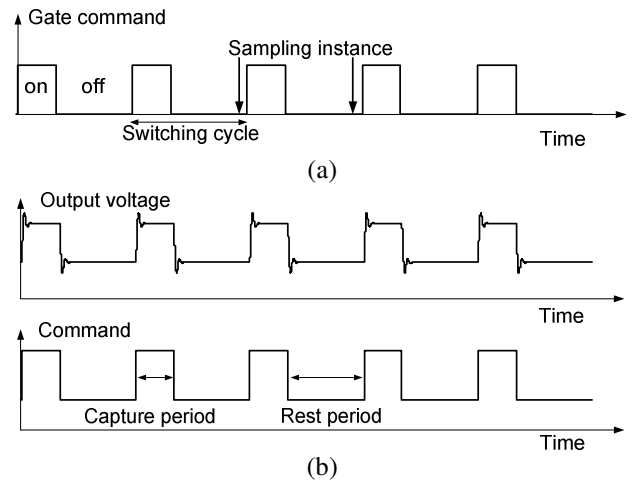


Figure 5. Data acquisition details. (a) Sampling instance in relation to gate command. (b) Capture and rest periods.

#### IV. EXPERIMENTAL

Two types of converters (Buck and Boost) were used to evaluate the proposed identification method. The synchronous data acquisition was implemented digitally on a TMS320F2407 DSP evaluation board [12], [13]. The step response were captured by a 10 bits A/D (3mV/bit), saved in local RAM and at the end of measuring sequence (repeated step injections) transmitted to a PC for off-line processing in MATLAB on a PC.

For both converters, the input voltage was 10V. Sensing gain was 1/7 (yielding 21mV/bit resolution of the output sensing). The switching frequency and sampling rate were 50KHz.

The parameters of the Buck stage were:  $L=75\mu\text{H}$  ( $R_L=250\text{m}\Omega$ ),  $C=100\mu\text{F}$  ( $\text{ESR}=300\text{m}\Omega$ ), load resistance:  $5\Omega$ , switch-on resistance (IRF640):  $0.18\Omega$ , diode forward voltage (1N5822): 0.5V. Duty cycle step was: 0.1 to 0.5; sequence length: 200 data points (= switching cycles); number of repeated sequences: 5; rest period between sequences: 500 switching cycles.

The parameters of the Boost stage were:  $L=1400\mu\text{H}$  ( $R_L=350\text{m}\Omega$ ),  $C=100\mu\text{F}$  ( $\text{ESR}=300\text{m}\Omega$ ), load resistance:  $50\Omega$ , switch-on resistance (IRF640):  $0.18\Omega$ , diode forward voltage (1N5822): 0.5V. Step values: 0.1 to 0.5; sequence length: 500 data points; number of repeated sequences: 5; rest period between sequences: 1000 switching cycles. The parameters of this experiment were chosen to emphasize the RHP zero effect of the boost converter.

The identified discrete time Buck transfer function was found to be

$$A(z) \approx \frac{-0.00644z + 0.0088}{z^2 + 1.987z + 0.98} \quad (5)$$

and for the Boost stage

$$A(z) \approx \frac{0.1414z + 0.047}{z^2 + 1.753z + 0.803} \quad (6)$$

Figs. 6 and 7 show the results of the average of the measured sequences and the step responses obtained from the identified buck and boost converters respectively. A further confirmation of the accuracy of the identification method can be observed the comparing the frequency responses of the identified converters and the one obtained using average simulation (Figs. 8 and 9).

#### V. DISCUSSION AND CONCLUSIONS

A time-domain based identification method for PWM converters was developed and verified by simulations and experimentally. The proposed approach overcomes the effects of switching and quantization noise by averaging the results of repeated perturbation sequences. It was found that the optimum number of data sequences is 5 since no improvement was observed when adding a larger number of sequences.

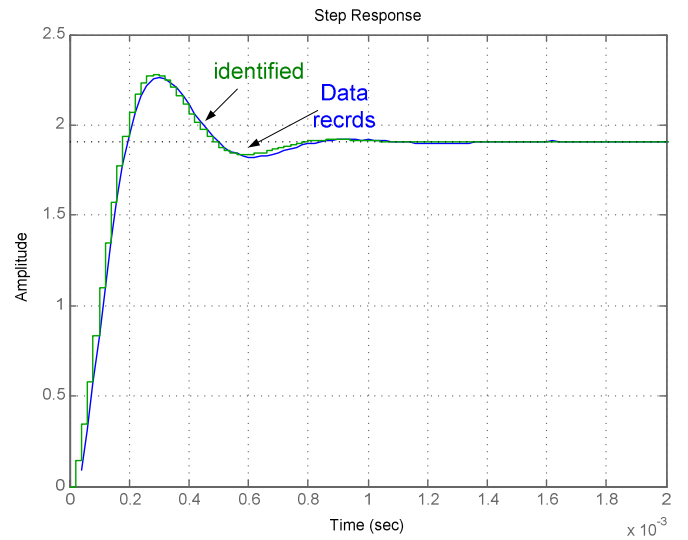


Figure. 6. Step response of the identified Buck converter (Eq. 5) and original records obtained from the experimental system.

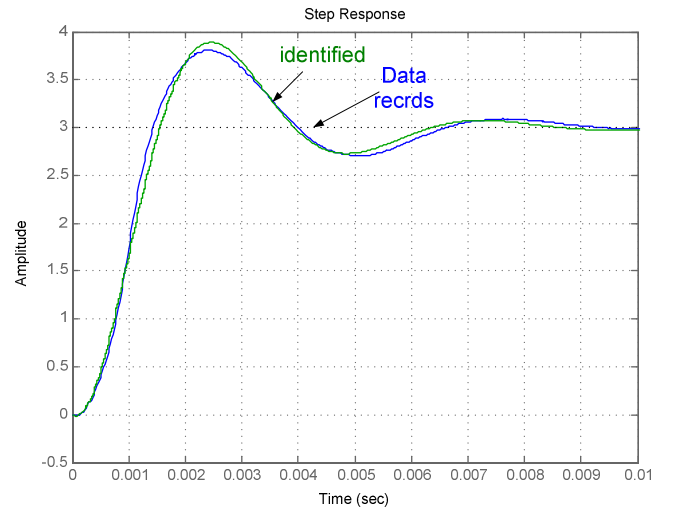


Figure. 7. Step response of the identified Boost converter (Eq. 6) and original records obtained from the experimental system.

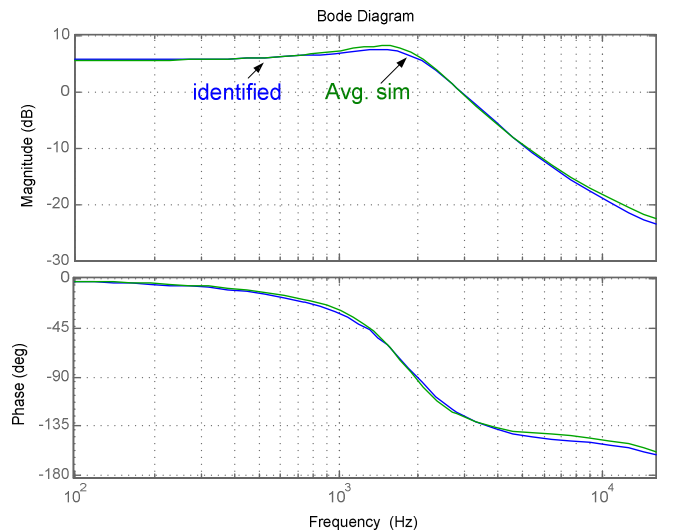


Figure. 8. Frequency response of the identified experimental Buck converter (Eq. 5) compared to the frequency response of average simulation.

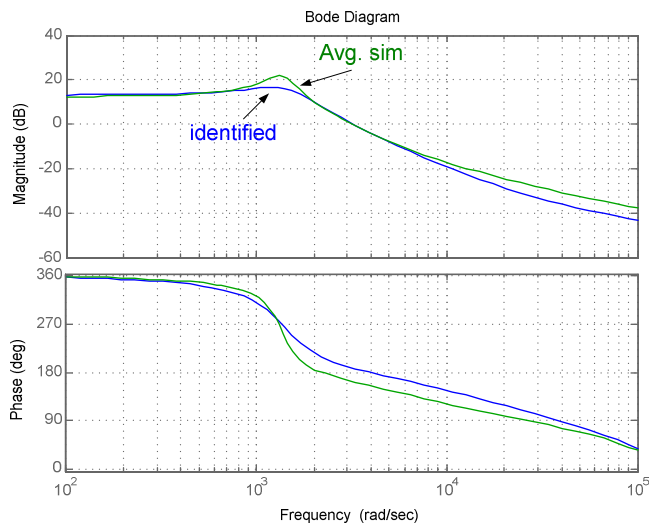


Figure. 9. Frequency response of the identified experimental Boost converter (Eq. 6) compared to frequency response of average simulation.

The extracted, template based, models were found to reproduce faithfully the response of a Buck and Boost converters.

A qualitative comparison in terms of memory efficiency and data acquisition time suggests that the proposed identification method is advantageous over frequency-domain based approaches. For instance, in order to properly identify the illustrated Buck type stage, by cross-correlation technique, for the same switching frequency and a modest measurement bandwidth resolution of 50 Hz, in the presence of noise (switching and quantization), a 10-bit random sequence is needed for the excitation and at least three repetitive sequences [5]. This translates into approximately 3000 data cells for the recording of the output signals and equivalent number of cells for the excitation. On the other hand, using the proposed approach, as reported in this study, we utilized only 1000 data cells for the output records, and since we applied a simple step perturbation of the duty command, only five additional synchronization cells to flag the start-of-sequence are needed. That is, the capturing unit would require one-sixth of the memory capacity and the acquisition time is reduced by two-thirds. Obviously, this difference will be substantially larger when a finer bandwidth resolution is attempted.

The advantages of the proposed identification method are: small number of data samples, a straightforward, and one stage, procedure with no need for further data manipulation or transformation, and using a generic template for all PWM topologies. These attributes makes it practical for implementation in on-line identification applications.

#### ACKNOWLEDGMENT

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